Black – Multiply Decimals with Decimals

Scientific Notation

Writing and Evaluating Scientific Notation

Scientific notation is a shorthand way of writing numbers using powers of 10. You write a number in scientific notation as the product of two factors.

Second factor is a power of 10.

$7{,}500{,}000{,}000{,}000 = 7.5 \times 10^{12}$

First factor is greater than or equal to 1, but less than 10.

Scientific notation lets you know the size of a number without having to count digits. For example, if the exponent of 10 is 6, the number is in the millions. If the exponent is 9, the number is in the billions.

1. Complete the chart below.

$5 \times 10^4 = 5 \times 10.000 = 50.000$
$5 \times 10^3 = 5 \times 1000 =$ ☐
$5 \times 10^2 = 5 \times$ ☐ = ☐
$5 \times 10^1 = 5 \times$ ☐ = ☐
$5 \times 10^0 = 5 \times$ ☐ = ☐
$5 \times 10^{-1} = 5 \times \frac{1}{10} = 5 \times 0.1 = 0.5$
$5 \times 10^{-2} = 5 \times$ ☐ $= 5 \times 0.01 = 0.05$
$5 \times 10^{-3} = 5 \times$ ☐ $= 5 \times$ ☐ $= 0.005$
$5 \times 10^{-4} = 5 \times$ ☐ $= 5 \times$ ☐ = ☐

2. Patterns. Describe the pattern you see in your chart.

3. a. Based on the pattern you see, simplify 5×10^7

b. Simplify 5×10^{-6}.

Multiplying and Dividing Powers of 10

Multiplying powers of 10 simply requires adding exponents.
For example:

$10^4 \times 10^7 = \underbrace{10{,}000}_{10^4} \times \underbrace{10{,}000{,}000}_{10^7} = \underbrace{100{,}000{,}000{,}000}_{10^{4+7} = 10^{11}}$

$10^5 \times 10^{-3} = \underbrace{100{,}000}_{10^5} \times \underbrace{0.001}_{10^{-3}} = \underbrace{100}_{= 10^{5+(-3)} = 10^2}$

$10^{-8} \times 10^{-5} = \underbrace{0.00000001}_{10^{-8}} \times \underbrace{0.00001}_{10^{-5}} = \underbrace{0.0000000000001}_{10^{-13}}$

Dividing powers of 10 requires subtracting exponents.
For example:

$\frac{10^5}{10^3} = \underbrace{100{,}00}_{10^5} \div \underbrace{1{,}000}_{10^3} = \underbrace{100}_{= 10^{5-3} = 10^2}$

$\frac{10^3}{10^7} = \underbrace{1{,}000}_{10^3} \div \underbrace{10{,}000{,}000}_{10^7} = \underbrace{0.0001}_{= 10^{3-7} = 10^{-4}}$

$\frac{10^{-4}}{10^{-6}} = \underbrace{0.0001}_{10^{-4}} \div \underbrace{0.000001}_{10^{-6}} = \underbrace{100}_{= 10^{-4-(-6)} = 10^2}$

Powers of Powers of 10

We can use the multiplication and division rules to raise powers of 10 to other powers. **For example:**

$(10^4)^3 = 10^4 \times 10^4 \times 10^4 = 10^{4+4+4} = 10^{12}$

Note that we get the same result by simply multiplying the two powers:

$(10^4)^3 = 10^{4 \times 3} = 10^{12}$

Adding and Subtracting Powers of 10

There is no shortcut for adding or subtracting powers of 10, as there is for multiplication or division. The values must be written in longhand notation. **For example:**

$10^6 + 10^2 = 1{,}000{,}000 + 100 = 1{,}000{,}100$

$10^8 + 10^{-3} = 100{,}000{,}000 + 0.001 = 100{,}000{,}000.001$

$10^7 - 10^3 = 10{,}000{,}000 - 1{,}000 = 9{,}999{,}000$

In scientific notation, you use a negative exponent to write a number between 0 and 1.

Example 2
Write 0.000079 in scientific notation.

0.000079	Move the decimal point to get a decimal greater than 1 but less than 10.
5 places	
7.9	Drop the zeros before the 7.
7.9×10^{-5}	The decimal point moved 5 places to the right.
	Use -5 as the exponent of 10.

Write each number in scientific notation.

4. 0.00021 **5.** 0.00000005 **6.** 0.0000000000803

You can change expressions from scientific notation to **standard form** by simplifying the product of the two factors.

Example 3
Write each number in standard form.

a. 8.9×10^{5}		**b.** 2.71×10^{-6}
8.90000	Add zeros while moving the decimal point.	000002.71
890,000	Rewrite in standard form.	0.00000271

Write each number in standard form.

7. 3.21×10^{7} **8.** 5.9×10^{-8} **9.** 1.006×10^{10}

Write each decimal in scientific notation.

10. 0.0001 **11.** 0.000007 **12.** 0.6

13. Write these numbers in standard form.

a. 4.28×10^{-1}
b. 6.7×10^{-4}
c. 9.144×10^{-3}
d. 1.3879×10^{-2}
e. 4.29×10^{-7}
f. 8×10^{-5}

14. Match the letters (a-h) in the first column of the following table with the numbers (1-8) in the second column.

Scientific Notation	Standard form
a. 1.032×10^{2}	**1** 0.0001032
b. 1.032×10^{-3}	**2** 103200
c. 1.032×10^{5}	**3** 0.1032
d. 1.032×10^{1}	**4** 103.2
e. 1.032×10^{-1}	**5** 1032
f. 1.032×10^{-4}	**6** 10.32
g. 1.032×10^{0}	**7** 1.032
h. 1.032×10^{3}	**8** 0.001032

15. Write these numbers in Scientific Notation.

a. 76.8
b. 7680
c. 0.00768
d. 7.68
e. 76,800,000
f. 0.0000000768

16. Renata uses her calculator to work out 458^{9}. The display shows 8.867257127 23. Write this number in

a. scientific notation
b. standard form

17. Kyo-Chung uses his calculator to work out $\frac{0.000\,034\,7}{897\,000\,000}$. The display shows 3.86845039 – 14. Write this number in

a. scientific notation
b. standard form

18. Work out these questions using a calculator. Write your answer in scientific notation.

a. 80,000,000 x 300,000
b. $478{,}000^{2}$
c. 52^{12}
d. 0.000005 ÷ 800,000

19. Here are three numbers written in scientific notation. Which is the largest?

a. 6.78×10^{-4}
b. 9.3×10^{-5}
c. 8.2×10^{-4}

20. Arrange these numbers in order from smallest to largest, writing them in standard form.

a. 3.9×10^{-5}

b. $2.18\ 10^{4}$

c. 5×10^{1}

d. 1.032×10^{-2}

21. The lightest of all atoms, hydrogen, has a diameter of 1×10^{-8} cm and weighs 1.7×10^{-24} grams. Write these measurements as ordinary numbers.

22. A computer works with numbers in *normalized floating point form*. An example is 0.46730218×10^{4}. Write this number in both *scientific notation* and *standard form.*

Star Travel

Solve. Write your answer in scientific notation unless otherwise directed. Use the information in early problems to help find later solutions…

23. An unmanned spacecraft sets out to explore the moon, Jupiter, and Alpha Centauri, the closest star in our galaxy. A typical rocket travels about 20,000 mi/h. Write this number in scientific notation.

24. The trip to the moon will take about 12 h. Use your answer to Exercise 23 and the formula $d = rt$ to find the distance to the moon in scientific notation.

25. The trip from the moon to Jupiter will take about 24,000 h.

a. Write the number of hours in scientific notation.

b. Find the distance from the moon to Jupiter.

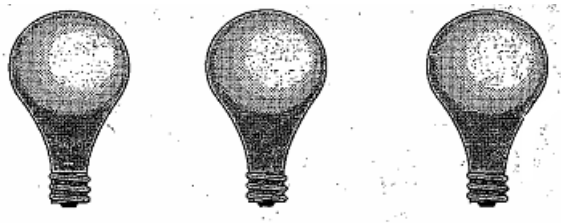
c. Write the number of days the journey will take in standard form. (1day = 24h).

26. From Earth, the trip to Alpha Centauri will take about 1.25×10^{9} h. Find the distance to Alpha Centauri.

27. The most distant star in the Milky Way in about 2.5×10^{4} times as far from Earth as Alpha Centauri. Find the distance to this star.

28. Name at least three real-life quantities which are conveniently written in scientific notation.

29. Ten 100-watt light bulbs use 1 kilowatt of electricity per hour. If electricity cost 8.4 cents per kilowatt, how much does it cost when a 40-watt, a 75-watt, and a 100-watt bulb are on for 8 hours?



30. Applying Decimal Operations

Lost in Space

I'm sure you've all heard about the costly mix-up resulting in the loss of NASA's Mars Climate Orbiter in late September of 1999. It seems the engineers in Colorado were working with English units (aka Imperial or Customary units) and the engineers in California were working with metric units. Neither group caught the discrepancy!

This is a pretty hard lesson about how important units are, especially considering the spacecraft was worth about $125 million. Jeepers! I'd hate to have to pay for that out of my allowance.

Let's take a look at this error on a smaller scale. Suppose the engineers in Colorado designed a square panel that was one yard by one yard, but the engineers in California thought the panel was one meter by one meter when they constructed it.

What is the difference, *expressed in metric units,* in the areas of the two panels? You will have to find a unit conversion between yards and meters to solve this!

Bonus: Express this difference as a percentage of the smaller panel (i.e., the bigger panel is what percent larger than the smaller panel?).

31. Irish Specials

Last Wednesday my family and our Irish friend, Moira, went out for dinner to celebrate St. Patrick's Day. As we entered the restaurant, we saw this sign:

The menu included the following items:

Cooked cabbage	$1.25
Irish potatoes	$1.75
French fries	$2.25

Shepherd's pie	$7.75
Cheeseburger	$5.75
soda bread	$1.50
milk	$1.50
juices	$2.25
iced tea	$1.25
decaf coffee	$1.50
slice of pie	$3.00

Moira, despite being Irish, was not much in the mood for Irish food (though she loves soda bread). The rest of us were up for a big Irish meal. Here is what each of us ordered:

Lisa	**Frank**	**T.J.**	**Moira**
cooked cabbage soda bread shepherd's pie milk grasshopper pie decaf coffee	cooked cabbage soda bread shepherd's pie ice tea key lime pie	milk Irish potatoes tastes of Mommy's and Daddy's	soda bread cheese burger French fries cranberry juice chocolate pie decaf coffee

Sales tax where I live is 4.5%, so I need you to help me calculate our final bill. Don't forget all the specials!

Bonus: Estimate the tip we should leave if we receive excellent service. A tip for excellent service is usually 15% to 20% of the food bill before tax.

32. Cavity-Less Caper

Daisy Dentifriss is very concerned about her teeth and takes very good care of them. She brushes twice a day and flosses once a day as her dentist has shown her. Now that she is on her own and must supply herself with the things she needs to keep her teeth bright, beautiful, and forever in her mouth, she is wondering about the cost of her floss. Daisy is now twenty years old, just out of college and supporting herself. She figures that she uses about 18 inches of dental floss every time she flosses her pearly whites with waxed floss, but when she used unwaxed, she uses about 22 inches. Every 6 months she visits her dentist for a check up. She gives Daisy a free package of dental floss containing 50 yards (waxed cinnamon flavored). Her mother, who is now 45 years old, puts a container of floss of 100 yards (unwaxed) in her birthday package each year. In the store she finds many types from which to choose.

Unwaxed, 63 yards @ $1.19
Waxed, mint or cinnamon, 50 yards @ $1.09
Waxed, 100 yards @ $2.49
Waxed mint, 100 yards @ $ .99

Unwaxed or waxed, 100 yards @ $ 1.39
Waxed, 200 yards @ $3.79
Natural flossing ribbon, 30 yards @ $3.59(uses all natural ingredients including bees wax)

HOW MUCH MONEY WILL DAISY DENTIFRISS SPEND ON HER DENTAL FLOSS IN HER LIFE TIME?

Please record all choices you are considering along the way. How and why you made these decisions is also important for someone who is reading your paper. Remember you want to present this so it is readable, easy to understand and well documented with facts.

Solutions

1.
- 5000
- 5 x 100 = 500
- 5 x 10 = 50
- 5 x 1 = 5
- 5 x $\frac{1}{100}$
- 5 x $\frac{1}{1000}$ = 5 x 0.001
- 5 x $\frac{1}{10000}$ = 5 x 0.0001 = 0.0005

2. (Answers will vary)

3. **a.** 50,000,000
b. 0.000005

4. 2.1×10^{-4}

5. 5×10^{-8}

6. 8.03×10^{-11}

7. 32,100,000

8. 0.000000059

9. 10,060,000,000

10. 1×10^{-4}

11. 7×10^{-6}

12. 6×10^{-1}

13. **a.** 0.428
b. 0.00067
c. 0.009144
d. 0.013879
e. 0.000000429
f. 0.00008

14. **a.** 4
b. 8
c. 2
d. 6
e. 3
f. 1
g. 7
h. 5

15. **a.** 7.68×10^{1}
b. 7.68×10^{3}
c. 7.68×10^{-3}
d. 7.68×10^{0}
e. 7.68×10^{7}
f. 7.68×10^{-8}

16. **a.** $8.867257127 \times 10^{23}$
b. 886,725,712,700,000,000,000,000

17. **a.** $3.86845039 \times 10^{-14}$
b. 0.00000000000000386845039

18. **a.** 2.4×10^{13}
b. 2.28484×10^{11}
c. $3.908770065 \times 10^{20}$
d. 6.25×10^{-12}

19. 0.00082

20. 0.000039, 0.01032, 50, 21800

21. 0.00000001, 0.00000000000000000000000017

22. 4673,0218, 4.6730218×10^{3}

23. 2×10^{4} mi/h

24. 2.4×10^{5} mi

25. **a.** 2.4×10^{4} h
b. 4.8×10^{8} mi
c. 1,000 days

26. 2.5×10^{13} mi

27. 6.25×10^{17} mi

28. Answers will vary.

29. 14.45 cents
If ten 100 watt bulbs use 1 kilowatt, then one 100 watt bulb would use .1 kilowatts of electricity. Because a 75 watt bulb is .75 of a 100 watt bulb, a 75 watt bulb would use .75 x .1 kilowatts = .075 kilowatts of electricity.

A 40 watt bulb is .4 of a 100 watt bulb so it would use .4 x .1= .04 kilowatts of electricity.
.1 kilowatts + .075 kilowatts + .04 kilowatts = .215 kilowatts per hour
.215 kilowatts per hour x 8 hours =1.72 kilowatts
1.72. kilowatts x 8.4 cents =14.448

30. The unit conversion between meters and yards is 1 yard = 0.9144 meters, so the area of one square yard is 0.83612736m^2.

A) Since the area of a square meter is 1m^2, the difference between the two is 0.16387264m^2

B) To find the percent, divide the difference by the smaller area and multiply by 100: 0.16387264m^2/0.83612736m^2 = 0.1959900463*100 = about 19.6%. So the bigger panel is close to 20% larger than the smaller panel!!

31. The final bill is $47.54. A tip for excellent service is between $7.01 and $9.35.

I listed everything eaten and what they cost and added it up. I made a second list of the costs of things that were not Irish or green and added it up. The Irish Dinner, green milk, green grasshopper pie, green key lime pie, Irish potatoes, and soda bread were Irish or green.

		Food bill	Taxed
Lisa	Irish dinner	$8.50	
	milk	$1.50	
	gr. pie	$3.00	
	coffee	$1.50	$1.50
Frank	Irish dinner	$8.50	
	iced tea	$1.25	$1.25
	key lime pie	$3.00	
T.J.	milk	$1.50	
	Irish potatoes	$1.75	
Moira	soda bread	$1.50	
	cheeseburger	$5.75	$5.75
	French fries	$2.25	$2.25
	cranb. juice	$2.25	$2.25
	choc. pie	$3.00	$3.00
	coffee	$1.50	$1.50
		$46.75	$17.50

The food bill totaled $46.75 and the taxed items totaled $17.50. The tax was .045 x $17.50 = .7875 or $0.79 rounded. The final bill was $46.75 + $0.79 = $47.54.

Tip calculation
.15 x $46.75 = 7.0125 or $7.01 rounded.
.20 x $46.75 = $9.35

32. There are many possible solutions depending on the assumptions that students make, but while the mother is alive dental floss should cost nothing or only a few cents (depending on whether or not she has her teeth flossed the days she goes to the

dentist). Lifetime costs will depend on how long mom lives, whether the dentist continues to live and give her floss, and what type of floss she chooses to buy.

Expert

The student makes appropriate assumptions needed in solving the task.

The Cavity-Less Caper

I am assuming that Daisy will live for 40 years. She uses dental floss once every day. Her mother will live for 70 years.

The first thing that I am going to do is make a kind of chart and try to figure out how much it will cost for 200 yards of floss - every kind. Then I can figure out the best price

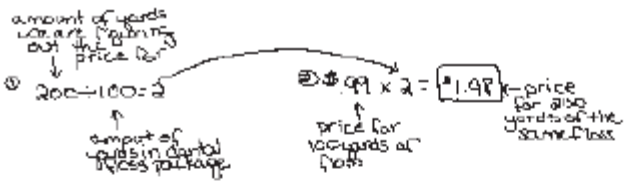
Kind of dental floss	Reg. Price	Approx. price for 200 yards
unwaxed - 65 yards	$1.19	$3.78
waxed mint or cinnamon - 60 yards	$1.09	$4.36
waxed - 100 yards	$2.49	$4.98
waxed mint - 100 yards	$.99	$1.98 - best price
unwaxed or waxed - 100 yards	$1.39	$2.78
waxed - 200 yards	$3.79	$3.79
Natural floss with 30 yards	$3.59	$23.93 WOW!

The student goes beyond the task requirements and determines the best buy.

To figure out how much 200 yards of dental floss would cost, I multiplied the original price by how many times the amount of yards would go into 200 yards.

Example: waxed mint - 100 yards. price - $.99

The student creates an accurate and appropriate math representation.



The student labels all of her/ his work.

I am assuming that Daisy uses waxed dental floss usually, but likes the unwaxed floss she gets at Christmas. I am also assuming that she wants the best priced dental floss.

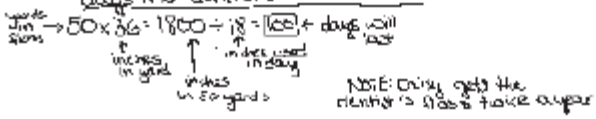
As we can see from the chart, the best priced dental floss (which is also waxed) is waxed mint - 100 yards. The price is $.99 for $1.98 for 200 yards).

The student communicates facts necessary in solving the problem.

Now, I am going to figure out how much dental floss (and the cost) Daisy will use in one year. First, I will figure out what she is given.

facts:
- Price of dental floss: 99¢ for 100 yards (waxed mint)
- amount of waxed floss used per day: 19 inches
- days in year: 365 days
- number of times one flosses: once per day
- number of inches in 1 yard: 36 inches
- twice a year, Daisy gets dental floss from her dentist - 50 yards, waxed, cinnamon
- once a year, her mother gives Daisy 100 yards, unwaxed.
- Daisy uses 22 inches of waxed floss.

days the dentist's floss will last:



NOTE: Daisy gets the mother's floss twice a year

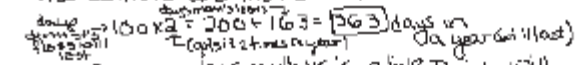
days mom's floss will last:

100 x 36 = 3600 ÷ 22 = 163 ← days will last

The student presents the solution in a logical sequential order.

Now we know how many days the dentist's and mother's floss will last. We have to remember that Daisy gets the dentist's floss twice a year, and the mothers once (while she is alive).

This is how many days in a year the dentist's and mother's floss will last:



So, when her mother is alive, Daisy will only have to pay about 1¢ a year for dental floss.

This is how much Daisy's dental floss will cost.

(turn page)

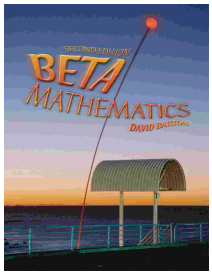
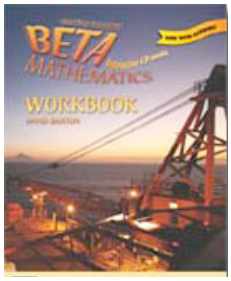
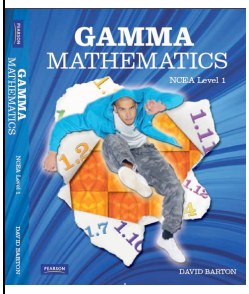
in her entire life!

Answer:
$26.78

P.S. This problem has a lot of little tricks. If Dad hadn't asked me how long the mother would live (which I had forgotten about) I wouldn't have had to go to lots of trouble!

Bibliography Information

Teachers attempted to cite the sources for the problems included in this problem set. In some cases, sources may not have been known.

Problems	Bibliography Information	
Examples and 4-22	Barton, David. Beta Mathematics. Pearson Education New Zealand.	
Examples and 4-22	Barton, David. Beta Mathematics Homework Book. ISBN 978-1-4425-0017-4. Pearson Education New Zealand, 2000.	
Examples and 4-22	Barton, David. Gamma Mathematics Pearson Education. New Zealand, 2000	
29-32	The Math Forum @ Drexel (http://mathforum.org/)	The Math Forum @ Drexel PEOPLE LEARNING MATH TOGETHER