2007
Chapter Competition
Sprint Round
Problems 1–30

Name ____________________________________________
School __________________________________________

DO NOT BEGIN UNTIL YOU ARE
INSTRUCTED TO DO SO.

This section of the competition consists of 30 problems.
You will have 40 minutes to complete all the problems.
You are not allowed to use calculators, books or other
aids during this round. Calculations may be done on
scratch paper. All answers must be complete, legible and
simplified to lowest terms. Record only final answers in the
blanks in the right-hand column of the competition booklet.
If you complete the problems before time is called, use the
remaining time to check your answers.

In each written round of the competition, the required unit
for the answer is included in the answer blank. The plural
form of the unit is always used, even if the answer appears
to require the singular form of the unit. The unit provided
in the answer blank is the only form of the answer that will
be accepted.

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<th>Total Correct</th>
<th>Scorer’s Initials</th>
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1. Triangle ABC is an obtuse, isosceles triangle. Angle A measures 20 degrees. What is the measure of the largest interior angle of triangle ABC?

2. Quentin spent $480 to purchase 30 books. Using the same average price per book, how much will 45 books cost?

3. In Clara county, 25% of households earn less than $30,000 per year, and 65% of households earn less than $80,000 per year. What is the largest possible percent of households that could earn between $30,000 and $80,000 per year?

4. For what value of \( x \) is the following equation true:
\[
3000 + x - 2000 = 1500 + 1000?
\]

5. How many cubic feet are in one cubic yard? One yard is equal to three feet.

6. Forty-eight congruent parallelograms with sides of length 62 feet and 20 feet are placed in a chevron pattern forming hexagon ABCDEF, as shown. What is the perimeter of hexagon ABCDEF?
7. The mean of four distinct positive integers is 5. If the largest integer is 13, what is the smallest integer?

8. Congruent segments are used to form equilateral triangles in this sequence so that each figure contains one more triangle than the preceding figure. Figure 3 of the sequence is made with seven congruent segments. How many congruent segments must be used to make Figure 25 of the sequence?

9. What is the sum of the odd integers from 11 through 39, inclusive?

10. The average amount of money spent by a person who attended a local sporting event in 2000 was $8.00, of which 75% was the ticket price. In 2005, the average amount of money spent by a person who attended a local sporting event increased by 50%, but the ticket price did not increase. By how many dollars did the non-ticket costs of 2000 increase to become the non-ticket costs of 2005?

11. The I-Pick-Up messenger service delivers packages using the following rate structure: $1 per ounce of the packages’ weight plus $5 for each distinct drop-off site. Then a 4% service fee per order is added to the subtotal of the weight and drop-off site charges. Chen Li places the order below. How much should I-Pick-Up charge for Chen Li’s order? (There are 16 ounces in one pound.)
   1 four-ounce package to Imatrin
   1 two-pound package to Imatrin
   1 eight-pound package to Storyville

12. A line contains the points (−1, 6), (6, k) and (20, 3). What is the value of k?
13. A particular convex polygon with seven sides has exactly one right angle. How many diagonals does this seven-sided polygon have?

14. The product of three consecutive odd integers is 1287. What is the sum of the three integers?

15. In square ABCD, point M is the midpoint of side AB and point N is the midpoint of side BC. What is the ratio of the area of triangle AMN to the area of square ABCD? Express your answer as a common fraction.

16. How many non-congruent triangles are there with sides of integer length having at least one side of length five units and having no side longer than five units?

17. What is the value of the following expression:

\[ \frac{1}{3} - \frac{1}{9} + \frac{1}{27} - \frac{1}{81} + \frac{1}{243} \] Express your answer as a common fraction.

18. A customer ordered 15 pieces of gourmet chocolate. The order can be packaged in small boxes that contain 1, 2 or 4 pieces of chocolate. Any box that is used must be full. How many different combinations of boxes can be used for the customer’s 15 chocolate pieces? One such combination to be included is to use seven 2-piece boxes and one 1-piece box.
19. The value of \([x]\) is the greatest integer less than or equal to \(x\). What is the arithmetic mean of the 10 members of the set 
\[\{[-\pi],[-\frac{1}{2}],[0],[\frac{1}{2}],[0.689],[\frac{\sqrt{2}}{2}],[\frac{\pi}{2}],[2],[\sqrt{5}],[\pi]\}\] 
Express your answer as a common fraction.

20. The summary of a survey of 100 students listed the following totals:
   - 59 students did math homework
   - 49 students did English homework
   - 42 students did science homework
   - 20 students did English and science homework
   - 29 students did science and math homework
   - 31 students did math and English homework
   - 12 students did math, science, and English homework
How many students did no math, no English, and no science homework?

21. Given that \(6x + y = 15\), the value of \(3x\) can be written in terms of \(y\) as \(ay + b\) for some numbers \(a\) and \(b\). What is the simplified value of \(a + b\)?

22. Suelyn counts up from 1 to 9, and then immediately counts down again to 1, and then back up to 9, and so on, alternately counting up and down (1,2,3,4,5,6,7,8,9,8,7,6,5,4,3,2,1,2,3,4,...). What is the 1000th integer in her list?

23. The positive three-digit integer \(N\) yields a perfect square when divided by 5. When \(N\) is divided by 4, the result is a perfect cube. What is the value of \(N\)?

24. Grady rides his bike 60% faster than his little brother Noah. If Grady rides 12 miles further than Noah in two hours, how fast does Noah ride?
25. The length of a diagonal of a square is \( \sqrt{2} + \sqrt{3} \) units. What is the area of the square? Express your answer in simplest form as \( \frac{a}{b} + \sqrt{c} \), where \( \frac{a}{b} \) is a common fraction and \( c \) has no perfect square factors other than 1.

26. Either increasing the radius or the height of a cylinder by six inches will result in the same volume. The original height of the cylinder is two inches. What is the original radius?

27. Consider this pattern where the positive, odd integers are arranged in a triangular formation. The 1st through 4th rows are shown; each row has one more entry than the previous row. What is the sum of the integers in the 15th row?

28. Four couples are at a party. Four people of the eight are randomly selected to win a prize. No person can win more than one prize. What is the probability that both members of at least one couple win a prize? Express your answer as a common fraction.

29. The points of this 3-by-3 grid are equally spaced horizontally and vertically. How many different sets of three points of this grid can be the three vertices of an isosceles triangle?

30. In parallelogram ABCD, \( AB = 16 \) cm, \( DA = 3\sqrt{2} \) cm, and sides \( AB \) and \( DA \) form a 45-degree interior angle. In isosceles trapezoid WXYZ with \( WX \neq YZ \), segment WX is the longer parallel side and has length 16 cm, and two interior angles each have a measure of 45 degrees. Trapezoid WXYZ has the same area as parallelogram ABCD. What is the length of segment YZ?