Black – Prime Factorization

Perfect!

A **proper factor** of a number is any factor of the number except the number itself. You can use proper factors to classify numbers.

A number is **abundant** if the sum of its proper factors is greater than the number itself.

Proper factors of 12: 1, 2, 3, 4, 6
1 + 2 + 3 + 4 + 6 = 16, and 16 > 12.
So, 12 is *abundant.*

A number is **deficient** if the sum of its proper factors is less than the number itself.

Proper factors of 16: 1, 2, 4, 8
1 + 2 + 4 + 8 = 15, and 15 < 16.
So, 16 is *deficient*.

Now you can probably guess the definition of a perfect number. A number is **perfect** if the sum of its proper factors is equal to the number itself.

Proper factors of 6: 1, 2, 3
1 + 2 + 3 = 6
So, 6 is *perfect*!

Tell whether each number is *abundant*, *deficient* or *perfect*.

1. 8

2. 9

3. 15

4. 18

5. 20

6. 24

7. 25

8. 28

9. 30

10. 35

11. What is the smallest counting number that is abundant?

12. Is it possible for a prime number to be perfect? Explain.

13. Is it possible for the sum of two deficient numbers to be an abundant number? Explain.

14. **CHALLENGE** Show why 496 is a perfect number.

Divisibility Rules

The divisibility rule for 13 is: Delete the last digit from the number, and subtract 9 times the deleted digit from the remaining number. If what is left is divisible by 13, then so is the original number.

15. Find the smallest four digits number such that when it is divided by 2, 3, 4, 5 or 6 the remainder is 1 each time but is completely divisible by 13.

16. Find the digit in the units place in the expansion of 3^{100}

17. Write a three digit number which is both a perfect square and a perfect cube.

18. There are four consecutive prime numbers written in ascending order. The product for the first three is 7429 and the last three is 12673. Find the numbers.

19. Fill in the blanks (with one digit) so that the statement is true:

a. ___928 is divisible by 13

The divisibility rule for 11 is: A number is divisible by 11 if the difference between the sum of the odd numbered digits (1st, 3rd, 5th…) and the sum of the even numbered digits (2nd, 4th…) is 0 or is divisible by 11.

20. Digit b is the units digit in the six-digit number 213,20b, which is divisible by 11. What is the value of b?

21. What is the smallest positive integer that has 2, 3, 4, 6, 7, and 12 as factors?

22. The number 100 can be written as the sum of a 1-digit prime number and a 2 digit prime number. What is the product of these prime numbers?

23. Summer Shoe Leftovers

The Swarthmore Shoe Emporium has 12 boxes of shoes left over from its summer stock. In order to get rid of them quickly, the store is having a summer leftover sale. The shoes come in three sizes: small, medium, and large. The shoes come in three styles: tennis shoes, flip-flops, and rain boots. Each style comes in three colors: blue, purple, and yellow.

To help them keep track once the sale starts, the Shoe Emporium employees have numbered the remaining boxes from one to twelve. With only twelve boxes left, not all combinations remain. However, the following clues should help you figure out just what's available:

All the prime-numbered boxes greater than 4 contain tennis shoes.
All even-numbered boxes less than or equal to 8 contain rain boots.
All the rest of the boxes contain flip-flops.
Boxes with numbers divisible by 3 contain large shoes.
Boxes with numbers that are factors of 88 contain medium shoes.
The remaining boxes contain small shoes.
All double-digit-numbered boxes contain blue shoes.
All odd, single-digit-numbered boxes contain purple shoes.
All other boxes contain yellow shoes.

Yen, Jill, and Karly plan to get to the store early so they will have first choice of the leftovers. Here are their particular wants and needs:

- Yen wants to buy yellow rain boots, but does not know her size.
- Jill wants some large, purple flip-flops.
- Karly knows that she doesn't want purple flip-flops, yellow rain boots, or tennis shoes.

Which boxes might satisfy each of the girls? Be sure that you include all of the possible boxes for each girl.

Bonus: Suppose that the store had exactly one pair of each style of shoe in each size and color for the leftover sale. How many boxes would they have?

24. Five Digits of Fun

My thousands digit is the first that is perfect,
But if you're not happy with that,
It's also the product of the first two primes,
According to my precocious cat.

My units digit is the odd of those primes,
And if you look at it and my tens,
You'll find a number divisible by three,
But not by the square of that, friends.

Now let's look at my hundreds and thousands.
If you examine those two as they're paired,
You'll find they form a number divisible by
An even prime that's been squared.

Now let's look at my body in whole.
Five different digits you see,
In order (or not) you'll find that they are
Divisible by nine, yes siree!

Can you find the mystery number?

25. Birthday Beach Bash

Twins Margaret and Dana and their friend, Rebecca, were celebrating their tenth birthdays with a party at the beach. The first fun activity was water games. As Chelsea arrived, she observed that 1/3 of the kids were playing with beach balls, 40% were exploring underwater with goggles, and .25 were floating in tubes.

The party continued with many games, presents, and the ever-popular cake and ice cream (though Isabelle thought they should have pie to celebrate Pi Day this week). When the party was over, each kid received a party bag and a balloon to take home. There were even three bags and balloons left for Margaret, Dana, and Rebecca.

How many balloons were needed for this birthday beach bash?

Bonus: If the first ten balloons cost one dollar each and additional balloon cost seventy-five cents each, how much was spent on balloons?

Number Patterns and Relationships

26. What experiments could you design that would test divisibility by 24, 30, or any other two-digit number? How do these experiments utilize number patterns?

27. The number 64 can be represented as the product of 16 and 4. What are some other ways to represent 64? What is the relationship among these representations?

28. The Math Committee has elected you to rename the term *exponent*. What will you call it? Give reasons for the new name.

29. "Every even number can be written as the sum of two primes." What are some ways you could prove or disprove this statement?

Goldbach's Conjecture

Christian Goldbach, an eighteenth-century Russian mathematician, believed that every even number greater than 4 could be written as the sum of two odd primes. For example, 16 = 5 + 11. He also believed that every even number greater than 4 could be written as the sum of three primes. For example, 16 = 2 + 3 + 11 or 16 = 2 + 7 + 7.

Complete the table.

Number	Sum of two primes		Sum of three primes	
10	**30.**		**31.**	
12	**32.**		**33.**	
14	**34.**		**35.**	
16	**36.**		**37.**	
18	**38.**		**39.**	
20	**40.**		**41.**	
40	**42.**		**43.**	
68	**44.**		**45.**	
94	**46.**		**47.**	
100	**48.**		**49.**	
144	**50.**		**51.**	

52. How Old is Dad?

On her birthday in 2003, Ellie's family sat at the dinner table together. Her little sister Anne asked their father how old he was. Dad replied, "I'm 3 years younger than your mother."

Not satisfied with that answer, Anne asked Mom how old she was. "If you five children add all your ages together, you will get my age," Mom said.

Uncle Ted wanted to help Anne figure it out, but he didn't remember the children's ages, so he asked them.

- Anne held up 4 fingers.
- Joe said, "I'm 2 years younger than Alex."
- Ellie replied, "I was born in 1985."
- "I am 1 year older than Joe," said Sara.
- Alex told him, "I'm half as old as Ellie."

Use the clues to find Dad's age. Be sure to explain how you found your answer and how you know you are right.

Extra: Curious Anne asked Uncle Ted how old he was. "Ah," he said, deciding to play their game. "My age is the product of two of yours, and it's a square number as well!"

How old is Uncle Ted? Be sure to tell how you know.

Solutions

1. Deficient

2. Deficient

3. Deficient

4. Abundant

5. Abundant

6. Abundant

7. Deficient

8. Perfect

9. Abundant

10. Deficient

11. 12

12. No; will always only add up to 1

13. Yes; Example 11 (deficient) + 9 (deficient) = 20 (abundant)

14. 496: 1, 2, 4, 8, 16, 31, 62, 124, 248

15. 1260

16. 1

17. 729

18. 17, 19, 23, 29

19. a. 5

20. b = 2

21. 84

22. 3.97 = 291

23.

Number of Box	Type of shoe	Size	Color
1	Flip flops	Medium	Purple
2	Rainboots	Medium	Yellow
3	Flip flops	Large	Purple
4	Rainboots	Medium	Yellow
5	Tennis	Small	Purple
6	Rainboots	Medium	Yellow
7	Tennis	Small	Purple
8	Rainboots	Medium	Yellow
9	Flip flops	Large	Purple
10	Flip flops	Small	Blue
11	Tennis	Medium	Blue
12	Flip flops	Large	Blue

The prime numbers that are greater than 4 but less than or equal to 12 are 5, 7, and 11. They would contain tennis shoes. The even numbers that are less than or equal to 8 are 2,4,6 and 8.
They contain rain boots.
The other boxes - 1,3,9, 10 and 12 contain flip flops.
The numbers divisible by 3 are 3,6,9, and 12. They contain large shoes.
The factors of 88 are 1,2,4,8, and 11. These contained medium shoes.
The other boxes - 5,7, and 10 contain small shoes.
The double digit numbers are 10,11, and 12. They contain blue shoes.
The odd, single-digited numbers are 1,3,5,7, and 9. They contain purple shoes.
The other boxes - 2,4,6,8 contain yellow shoes.

Yen wanted yellow rainboots. Looking on our chart we saw that they would be in boxes 2, 4, 6, and 8.

Jill wanted large purple flip flops. They are in boxes 3 and 9.

Karly wanted shoes that were not purple flip flops, so she would want blue or yellow flip flops. She did not want yellow rain boots so she could want purple or blue rain boots. But all the rain boots are yellow. She did not want any type of tennis shoe. There are blue flip flops in #10 and #12.

Bonus: We made a diagram. We know that we multiply when we are finding combinations.

Tennis shoes ----- Blue ----- large, medium and small
----- Purple ----- large, medium and small
----- Yellow ----- Large, medium and small

So there are 9 combinations for tennis shoes.
There are 9 more combinations for rain boots
And there are 9 more combinations for flip flops.
That would be 278 combinations all together.

There are three types of shoes, three colors and three sizes.
3 x 3 x 3 = 27

24. At least 4 numbers work here: 56403, 56493, 16803, and 16893 all meet the clues. First I found the thousands place, which has to be 6 (the first perfect number, since the factors 1,2, and 3 add up to 6). The units digit has to be 3 (the odd prime mentioned in the poem).

So, using X as a marker for missing digits (not as a variable), this is what I have:
X6XX3

Next, the ones and the tens have to create a number divisible by 3 but not 9 (the square of 3). Choices here for the tens and ones digit would be 03 or 93, since 63 would be divisible by 9.

So, X6X03 or X6X93

Next I worked on the hundreds and thousands. Those two digits have to start with a 6 and be divisible by 4 (the square of the only even prime number).

Choices here would be 60, 64, or 68. Since there are 5 different digits, the 0 can't be used twice. My choices now are:
X6093
X6403
X6493
X6803
X6893

Finally, I need the sum of the five digits to be divisible by 9 so that the five digit number can be divided by 9.

So, no digit would work for X6093 since a 9 would be required, and that would mean double 9's.

56403 has a sum of the digits equal to 18 (56403/9=6267)
56493 has a sum of the digits equal to 27 (56493/9=6277)
16803 has a sum of the digits equal to 18 (16803/9=1867)
16893 has a sum of the digits equal to 27 (16893/9=1877)

So, for an answer, 56403 fits the clues and can be the mystery number. Any of the others can be a bonus number.

25. They will need 63 balloons.
First, it says when Chelsea got to the party she saw one - third of the kids were playing with beach balls, so you know the total number of kids at the party is divisible by three. Then it says 40% were exploring with goggles. Since 40% is the same as 2/5, then the total number of kids must be divisible by 5. Then it says .25 were floating in tubes. This is the same as 1/4. This means that the total number of kids is divisible by 4. The lowest number of kids that would be divisible by 3, 5, and 4 is 60. You can get this by multiplying 3 x 5 x 4 to get 60. Therefore the number of kids is 60 plus the three birthday girls.

BONUS: They need 63 balloons. The first 10 cost $1.00 each for a total of $10.00. The rest cost $.75 each. There are 53 of those for a total of $39.75. For all 63 balloons, the cost would be $49.75.

My favorite balloon color is purple.

26. Answers will vary. Example: "To test for divisibility by 24, you can see if the number you're checking is divisible by 2, 3, and 4 (since 2 x 3 x 4 = 24). If the number is divisible by all those factors of 24, then it is also divisible by 24."

27. Answers will vary. Possible answers: 32 and 2; 8, 4, and 2, etc. The relationship between the factors is that they are all powers of 2.

28. Answers will vary

29. Answers will vary. For example, one way to disprove it would be to find a single instance where the sum of 2 primes does not equal an even number.

Number	Sum of two primes		Sum of three primes	
10	**30.**		**31.**	
12	**32.**		**33.**	
14	**34.**		**35.**	
16	**36.**		**37.**	
18	**38.**		**29.**	
20	**40.**		**41.**	
40	**42.**		**43.**	
68	**44.**		**45.**	
94	**46.**		**47.**	
100	**48.**		**49.**	
144	**50.**		**51.**	

52. Anne's dad's age is 43 years old. Uncle Ted's age is 36 years old.

To find out how old Anne's dad was I first had to find out the age of The children. I first had to find out Ellie's age. To do this I took 2003(present year) -1985 (the year she was born) = 18 years old.
I then figured Alex's age to be 9 by division 18 divided by 2=9.
To double check my work I then added 9+9=18.
I then moved on to Joe's age he said he was two years younger than Alex 9 – 2 = 7.

Then came Sara at one year older than Joe 7 + 1 = 8.
To figure out the father's age I added all the children together 18 + 9 + 7 + 8 + 4=46 making mom 46 years old. Dad said he was three years younger than mom I used subtration to figure his age out 46 – 3 = 43.

Extra credit

I wasn't sure what a square number was so I went on the computer and looked up the meaning of square numbers. I also looked at the first 51 square numbers and than multiplied each of the children's ages to figure out which age came to be a square root given me the answer of 36.
4 x 7 = 32
4 x 8 = 36
4 x 9 = 38
4 x 18 = 72
7 x 8 = 56
7 x 9 = 63
7 x 18 = 126
8 x 9 = 72
8 x 18 = 144
9 x 18 = 162

when I saw that 36 was the lowest square number next to 144 years old I figured out he was 36 years old

Bibliography Information

Teachers attempted to cite the sources for the problems included in this problem set. In some cases, sources may not have been known.

Problems	Bibliography Information	
23-25, 52	The Math Forum @ Drexel (http://mathforum.org/)	