Black – Rates

1. A faucet leaks at a rate of 1 pint every 2 hours for 4 full days. How many gallons of water leaked from the faucet?

   Note: 1 gallon = 4 quarts and 2 pints = 1 quart

2. A car that is traveling 90 feet per second is traveling how many miles per hour? There are 5280 feet in 1 mile. Express your answer to the nearest whole number.

3. Howard ran a half-marathon (13.1 miles) in 2 hours. He completed the first 10 miles in 95 minutes. How many minutes per mile did Howard average for the remainder of the race? Express your answer as a decimal to the nearest tenth.

4. The speed of light is 670,000,000 miles per hour. How many seconds does it take light to travel 121,000,000 miles? Express your answer to the nearest second.

5. If Andy bikes at a steady rate of 15 miles per hour, how many miles will he bike in four minutes?

6. What is the angular speed of the big hand on a clock, in degrees per minute?

7. Speeds at sea and in the air, are often measured in knots. The speed of one knot means one nautical mile per hour. A nautical mile is a distance of approximately 1.85 km.
   a. What is the speed in km/h of a 15 knot wind?
   b. A ship steams 30 nautical miles at a speed of 12 knots, then a further 20 nautical miles at a speed of 15 knots. What is the total time taken?
   c. How far, in kilometers, would a plane fly at a speed of 240 knots, between 10:30 am and 1:15 pm?
   d. A boat sails 850 m in 6 minutes. What is the equivalent speed in knots? Answer correct to 1 decimal place.

8. Julia jogs at a rate of six miles per hour. What is her rate in feet per second? Express your answer as a decimal to the nearest tenth.
   Note: There are 5280 ft in 1 mile.

9. While driving down the Aegean coast you see a sign that says gasoline costs 229,704 Turkish Lira per Liter. The current exchange rate is 410,000 TL to a United States dollar. How much does the gasoline cost in dollars per gallon?
   (1 gal ≈ 3.784 L) Round to the nearest cent.

10. Using A,B,C,D and E, list the corresponding speeds in order from fastest to slowest. Use 1 mile =5280ft, 1 km=0.62 miles and 1 m= 3.28 ft.

    a. 10,000 ft/min **b.** 60 miles/hr **c.** 100 ft/sec **d.** 100 km/hr **e.** 30 m/sec

11. For each of the first 200 units of a calculator a salesman sells, he gets a commission of $0.80. For each of the next 200 units he sells, he gets a commission of $0.90. For each subsequent unit he sells thereafter, he gets a commission of $1. How many units must he sell to earn a total commission of $420?

12. The table below shows the annual rates of interest paid on a bank's fixed deposits.

For the first $50,000	4%
For the next $50,000	3%
For the next $100,000	2%
For any additional amount exceeding $200,000	1%

A man had a fixed deposit account in the bank. If he received $6000 interest last year, what was his principal sum in his fixed deposit account?

13. The table shows the rates of personal income tax in a certain country.

Annual Income	Income tax
First $7500	Not taxable
Next $5000	2% per dollar above $7500
Next $10,000	3% per dollar above $12,500
Next $10,000	5% per dollar above $22,500
Next $50,000	10% per dollar above $32,500

Mr. Lee's annual incomes for last year and this year were $20,000 and $35,000 respectively. How much more income tax did he pay this year than last year?

14. An "All You Can Eat" buffet charged $14.90 per adult and $11.90 per child. It also charged $10 per kilogram of food, or part thereof, wasted. The bill for a table of 6 adults and 2 children was $143.20. What was the maximum possible weight of food these people wasted?

15. The table shows the monthly rates of subscription to an Internet service.

Package A	$10.25 for the first 12 hours $2.02 for each subsequent hour or part thereof
Package B	$25.70 for the first 24 hours $2.02 for each subsequent hour or part thereof
Package C	$103.00 for unlimited number of hours

a. Andrew used the service for 36 hours 25 minutes last month. How much more would he have to pay under package A than package B?
b. Michelle used the service for 14 hours 6 minutes last month. How much less would she have to pay under package A than package C?

16. A transport company charged $2 for each vase safely delivered but paid a penalty of $62 for each vase delivered broken or lost. If it collected $336 for the delivery of 392 vases, how many vases did it deliver broken or lost?

17. 3 lb of grade A ground coffee, 4 lb of grade B ground coffee and 5 lb of grade C ground coffee were mixed to form grade X ground coffee. The costs of grades A, B and C ground coffee, per pound, were $9, $6 and $3 respectively. Find the cost of 5 lb of grade X ground coffee.

18. Building Barns

The workers at Amy's Barn Company usually work in teams of 5. It takes a crew of 5 workers 9 days to build a barn.

A crew lost two of its workers. How many days should it take the remaining 3 workers to build a barn?

Extra: If a customer needs to have a barn built in 7 days, what is the minimum number of workers that Amy would need to put on the job?

It would take 15 days for 3 workers to build a barn. Extra: It would tale 7 workers to complete the barn in 7 days.

19. Movie Line Madness

Penelope sells tickets at the local movie theater. She is one of the fastest on the job, completing 100 transactions per hour. Odysseus is also pretty quick, completing 80 transactions per hour.

At three o'clock one afternoon there are 200 people in line, but on average only half the people in line are making transactions (the rest are family members, friends, etc.).

If Penelope starts selling tickets by herself, with Odysseus joining her at quarter after three, what time will they finish helping these 200 people?

Solutions

1. If the faucet leaks at a rate of 1 pint every 2 hours, then it leaks 12 pints in 1 day, and it would leak 48 pints in 4 days. There are 4 quarts in each gallon, and 2 pints in a each quart. Thus, there are 8 pints in one gallon. Consequently, 48 pints is 48 ÷ 8 = 6 gallons, so the faucet leaks 6 gallons of water in 4 days.

2. There are 60 x 60 = 3600 seconds in an hour, so if the car traveled 90 feet per second for an hour, it would go 90 x 3600= 324,000 feet. There are 5280 feet in one mile, so this is 324,000 ÷ 5280 = 61 mph, to the nearest whole number.

3. If Howard ran the 13.1 miles in 2 hours, then he ran 13.1 miles in 120 minutes. The first 10 miles took 95 minutes. That leaves 13.1 – 10 = 3.1 miles to be run in 120 – 95 = 25 minutes. The average pace is 25 ÷ 3.1 = 8.1 minutes per mile.

4. First of all, we know that it will take less than one hour for light to go 121,000,000 miles since it ravels 670,000,000 miles in a full hour. Dividing 121 by 670, we get

0.1806 hours or 0.1806 x 60 = 10,836 minutes or 10,836 x 60 ≈ 650 seconds.

5. One way to approach this problem is to see it as "Rate x Time = Distance" problem. We know the rate is 15 mph, and he is biking for four minutes, which is $\frac{4}{60}=\frac{1}{15}$ of an hour. (Remember that if our rate is in miles per hour, then our time must be in hours, and our distance will be in miles.) Now we can calculate that 15 mph x $\frac{1}{15}$ hr = 1 mile.

We also could view this as a proportional reasoning problem. We know that he travels 15 miles in 60 minutes, and we want to know the number of miles he travels in four minutes. This leads to the proportion $\frac{15}{60}=\frac{x}{4}$. Setting the cross products equal, we have $60x = 4(15)$. Dividing both sides by 60 then gets us to $x=\frac{4(15)}{60}=\frac{60}{60}=1\,\text{mile}$.

6. 6 deg/min

7. **a.** 27.75 km/h **b.** 3h 50 min

c. 1221 km **d.** 4.6 knots

8. Let's agree first that (1) multiplying an expression by 1 does not change the value of the expression and (2) a ratio is equal to 1 if the value in the numerator is equal to the value in the denominator. Keeping these two facts in mind, consider 6 mph or the ratio $\frac{6\ miles}{1\ hour}$. We want to change this to $\frac{?\ feet}{?\ sec}$. Take a look at the product $\frac{6\ miles}{1\ hour}\bullet\frac{1\ hour}{60\ min}\bullet\frac{1\ min}{60\ sec}\bullet\frac{5280\ ft}{1\ mile}$. The first ratio is our 6 mph, and each of the next three ratios is equivalent to 1 since the value in the numerator of each ratio is equal to the value in its denominator. This ensures that the expression is still equivalent to 6 mph. " Cancelling" factors in the numerators with factors in the denominators is a common practice when we are multiplying fractions, and after performing the same type of procedure with the units in the expression above., we are left with $\frac{6\bullet 5280 ft}{60\bullet 60\,sec}$. Dividing out factors of 6, 10 and another 6 from the numerator and denominator, we have $\frac{88}{10}=8.8$ feet/second.

9. About $2.12/ gal

10. To compare the various speeds, we can convert them all to the same units of measure. Let's use feet per second. Speed A is 10,000 feet per minute, which is 10,000 ÷ 60 ≈166.7 feet per second. Speed B is 60 miles per hour, which is (60 x 5280) ÷ (60 x 60) = 88 feet per second. Speed C is 100 feet per second. Speed D is 100 kilometers per hour, which is (100 x 0.62 x5280) ÷ (60 x 60) ≈ 90.9 feet per second. Finally, speed E is 30 meters per second, which is 30 x 3.28 = 98.4 feet per second. Listing the speeds in order from fastest to slowest, we get ACEDB.

11. 480 **12.** $250,000 **13.** $825

14. 3 kg

15. a. $8.79
b. $86.69

16. 7

17. $27.50

18. It would take 15 days for 3 workers to build a barn. Extra: It would take 7 workers to complete the barn in 7 days.

First I made a chart, a 5 by 9 chart. The 5 rows going up are for each little share each worker contributes on making the barn. The 9 columns going across are for each little piece of progress that happens each day if each worker did the same amount of work. Then I multiplied 5 by 9 to get the total amount of work days total for each person. 5 times 9 is 45 so if one person was building the house, then it would take 45 days to build. Next since 2 workers did not show up, I crossed out the 2 columns representing 2 workers because then the work will go on as if the workers were still there but they weren't I knew that they're was only 3 people left to work so 3 times 9 is 27 and 27 subtracted from 45 is 18. 18 needs to be divided by 3 because 18 is the number of work days that are left over and need to be used or else the barn won't be finished, so 18 divided by 3 is 6. 6 is the number of columns added to each person to make up for the people not showing up. 9 + 6 = 15 so 15 is the total days that the 3 workers need to work to finish the barn.

Extra: If the costumer wants it done in 7 days then 7 divided by 45 is 6 R3. (6 times 7 is 42 + 3 = 45)The workers have an extra 3 parts of the barn not finished and the barn needs to be finished so you round 6 up to 7.

19. Penelope and Odysseus will finish selling tickets at 3:40.

Penelope and Odysseus have 200 impatient people waiting in line. However, only half the people in line are buying tickets (the rest are accompanying the ticket-buyers.) Therefore, only 100 transactions need to be made. Penelope can make 100 transactions per hour, while Odysseus can make only 80. If Penelope starts selling at 3:00, by the time Odysseus joins her 15 minutes later she will have already sold 25 tickets. We figured this out because 15 minutes is 1/4 of an hour, and 25 tickets is 1/4 of 100 tickets. As you know, Penelope can sell 100 tickets per hour. When Odysseus joins her at 3:15, there are only 75 tickets left to sell. Over the next 15 minutes Penelope sells another 25 tickets and using the 1/4 equation, we found that Odysseus will sell 20 tickets. This means that at 3:30 together they will have sold 45 more tickets leaving them with only 30 tickets still to sell. Because in 15 minutes they can sell 45 tickets, and only 30 tickets are left, we went down 5 minutes to 10 minutes. Now all that was left was to figure out how many transactions they each could make in 10 minutes. We divided how many tickets they could sell together in an hour (180) by 6 because there are 6 tens in 60 which was 30. That was all the tickets they needed to sell. So they went off to their soda and popcorn break at 3:40.

Bibliography Information

Teachers attempted to cite the sources for the problems included in this problem set. In some cases, sources may not have been known.

Problems	Bibliography Information	
18, 19	The Math Forum @ Drexel (http://mathforum.org/)	
1, 3-5, 8-10	Math Counts (http://mathcounts.org)	
2, 6-7	Thomas, Alice, and Margaret Jordan. Maths in the Fast Lane. Putney: Phoenix Education, 1998. Print	