Black – Surface Area and Volume

(Note: when converting between length, volume, and mass, 1 cm^3 equals 1 mL^3, and 1 mL^3 equals 1 gram)

1. A rectangular container, 25 cm long, 18 cm wide, and 10 cm high, contains 4 L 320 ml of water. Another rectangular container, 10 cm long, 6 cm wide and 4 cm high is completely filled with water. If water from the second container is poured into the first container, how much water will overflow?

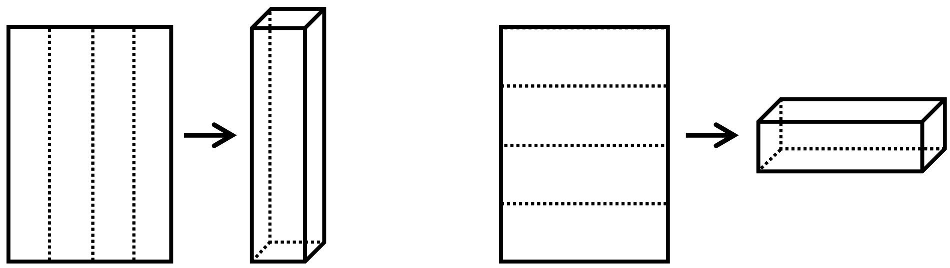
2. A painted 2 x 2 x 2 cube is cut into 8 unit cubes. What fraction of the total surface area of the 8 small cubes is painted?

3. Based on what you know about the volume of a sphere, write a formula for the volume of a hemisphere.

4. A natural gas pipeline ruptured, triggering an explosion. The amount of soil required to fill the hole made by the explosion was equivalent to the amount of soil in a rectangular prism 86 feet long, 46 feet wide and 21 feet deep. How many dump truck loads, of 20 cubic yards of soil each, were needed to transport the soil? (3 feet = 1 yard)

5. Let S = # square units in the surface area of a cube. Let V = # of cubic units in the volume of the cube. When the side length of the cube is doubled, by what factor is the ratio of S to V multiplied? Express your answer as a common fraction.

6. Take Two sheets of $8\frac{1}{2}$" by 11" paper. Fold one sheet vertically into fourths to form the sides of the rectangular prism. Fold the other sheet horizontally into fourths to form the sides of a different rectangular prism. How much more volume than the smaller prism does the larger prism have? Express your answer as a decimal to the nearest tenth.



7. A cylindrical quarter has a $\frac{15}{16}$ inch diameter and a $\frac{1}{16}$ inch height. What would be the number of inches in the height of a coin whose volume is exactly four times that of the given quarter and whose diameter equal $1\frac{1}{8}$ inches? Express your answer as a common fraction.

8. In a rectangular container completely filled with water and oil, there are 6480 cm^3 of water more than oil. If the container is 48 cm long, 27 cm wide and 18 cm high, find the depth of the water in the container.

9. The length and width of a rectangular container are 40 cm and 25 cm respectively. It is filled with sand and clay in the ratio 2 : 3. If the depth of the sand in the container is 6 cm, how much more clay than sand is there?

10. A rectangular container is filled $\frac{3}{4}$ with oil. The length, width and height of the container are in the ratio 7 : 5 : 3. If its length is 20 cm longer than its height, how much more oil can the container contain?

11. A rectangular tank, 72 cm long, 45 cm wide and 40 cm high, is filled with water to a depth of 17.9 cm. When two identical pieces of metal are added into the tank, the water level rises to 18.1 cm. Find the volume of each piece of metal.

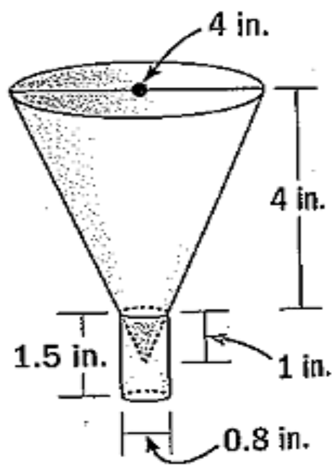
12. **Funnel** Most funnels consist of a cone whose tip is removed. The cone is then attached to a narrow cylinder. In the diagram below, the small cone inside the cylinder shows the portion of the large cone that has been cut off.

a) Find the volume of the large cone and the volume of the small cone to the nearest hundredth of a cubic inch. (note that the <u>radius</u> of the large cone is 4 in.)

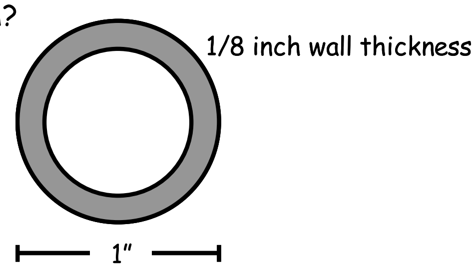
b) Calculate the difference between the volume of the large cone and the volume of the small cone.

c) Find the volume of the cylinder to the nearest hundredth of a cubic inch.

d) Use your results from part (b) and part (c) to find the total volume of the funnel to the nearest hundredth of a cubic inch.



13. What is the volume of water in a full 100-foot hose that has an outside diameter of one inch and a wall thickness of 1/8 inch?



14. A truck that weighs 6500 pounds when it is empty must travel over a bridge that has a weight limit of 40 tons. The truck has a rectangular box for hauling that is 25" x 12" x 8" tall. The truck will be hauling salt that weighs 35 pounds per cubic foot. To what depth can the salt be loaded in the box and still ensure that the weight limit of the bridge will not be exceeded? Remember that 1 ton = 2000 pounds.

15. Gold can pounded into very thin sheets. If a $1cm^3$ cube is pounded into a sheet exactly 1.25 cm wide and 20 cm long, how thick is the sheet?

16. The electricity to Jason's electric water heater is shut off because he needs to drain the tank to clean out mineral deposits. Jason now needs to refill his water heater. He has to be very sure that it is completely full of water before the electricity is turned back on because there are two heating elements in the heater that must be completely covered with water, or they will burn out and cause an expensive repair. The inside of the tank, which is a cylinder, measures 5 decimeters in diameter and is $1\frac{1}{2}$ meters high.

When Jason runs water, it fills a liter container in 6 seconds. It is now 6:35 P.M. and Jason has just turned on the water to fill the task. At what time will the tank be full so Jason can turn the electricity to the water heater back on. (Round to the nearest minute.)

17. Don's water heater is a cylinder that is 4 feet high and 1.5 feet in diameter. Don desperately wants to take a shower, but his water heater is broken. Don decided to try to heat water by placing garden hoses that are full of water out in the sun. The garden hoses have 3/4 inch inside diameters and Don wants to heat the same amount of water that he normally uses for a shower. During a typical shower, Don will use 1/10 the volume of the amount of water in his water heater. How many feet of hose should Don lay out in the sun? (Round your answer to the nearest ten.)

18. Can You Drink That Swimming Pool?

Steve was helping his dad fill their swimming pool with water. As usual, he had a lot of questions and his dad didn't know all the answers.

"Dad, do you think you could drink all that water?"

Dad responded, "I wouldn't want to, since it's got a lot of chlorine."

"But could you drink that much water?"

"No."

"Not even in a whole year?" Steve persisted.

Dad responded, "Well, there's a lot to think about - so let's do some estimating. The pool is about 12' by 24' and I'd say the average depth is about 4'6". I also know that there are about 7.48 gallons in a cubic foot and I drink about a quart of water a day. What do you think? Could you drink a pool's-worth of water in a year?"

"Gee, Dad, that is a lot to think about," said Steve.

"Well, let's start with a smaller estimation first. Our kiddie pool was about 5' by 5' and we'd fill it to about a foot deep. Approximately how long do you think it would take to drink that much water?" asked Dad.

What do you think?

1. Estimate how long it would take one person to drink the amount of water the kiddie pool held.

2. Estimate how long it would take one person to drink the amount of water in the larger pool.

Since you are estimating, we don't expect that everyone will have the exact same answers - but if you compare with your friends, your answers should be in the same ballpark. Also, remember that Steve and his dad were doing these calculations in their heads, so choose numbers that make the calculations easier to do mentally.

19. Filling the Freezer

The Math Club meets one day a week after school to do projects and math investigations. They enjoy having an ice cream snack at each meeting. Their project this week is to buy the ice cream they'll be storing in their freezer for their upcoming meetings.

Before they leave for the store they measure the freezer. The interior of the freezer is 65 cm wide, 45 cm deep, and 35 cm tall.

At the store they measure an ice cream container. The dimensions of the top of the box are 17.5 cm by 11.6 cm. The height is 12.2 cm.

One group, the Protractors, figures out how many boxes will fit in the freezer if they place each box with the lid facing up and the 17.5 cm dimension along the front of the freezer.

Another group, the Right Angles, agrees that they'll keep the lids facing upwards (to avoid messes as some of the containers start to get eaten) and they figure out how many boxes will fit if they turn the boxes and have the 11.6 cm dimension along the front of the freezer.

Questions: Which group can fit more ice cream boxes in the freezer? How much unused space would be left in the freezer if they buy that many boxes?

Extra: Describe a way to fit more whole containers of ice cream in the freezer. How many can you fit?

20. Splish Splash

The bathtub in our house is approximately twenty-three inches by fifty inches (these are inside measurements). When I fill the tub with water to a height of nine inches, I can completely submerge myself (except my head). When I do this, the height of the water rises to twelve inches.

When I put T.J. in the tub under the same conditions, the height of the water rises to nine and three-fifths inches.

What is the ratio of my volume to T.J.'s volume?

(Reminder - be sure to explain your process completely and show all your calculations.)

Solutions

1. 60 mL

2. Each of the eight cubes has three of its six faces painted, so ½ of the total surface area of the 8 small cubes is painted.

3. A hemisphere is half of a sphere, thus $V = \frac{2}{3}\pi r^3$

4. Multiplying 86 feet by 46 feet by 21 feet, we find that the volume of the prism is 83,076 cubic feet. Since the capacity of the dump trucks was given in cubic yards, we must either convert the volume of the prism into cubic yards or convert the capacity of the dump trucks into cubic feet. One cubic yard is 3 feet by 3 feet by 3 feet, which is 27 cubic feet. Dividing 83,076 by 27, we find that volume of the prism is 3076.9 cubic yards. Dividing this by 20 cubic yards for every dump truck, we find that it would take about 154 dump truck loads to fill the hole made by the explosion.

5. When the side length of a cube is doubled, the area is 4 times as great $2^2 = 4$' and the volume is 8 times as great $2^3 = 8$'. The ratio S to V is 4 to 8, or ½.

6. When we fold the sheet of paper vertically into fourths, we get a tall, skinny prism with a height of 11 inches and a square base measuring 8.5 ÷ 4 = 2.125 inches on a side. The volume of this tall, skinny prism is 2.125 x 2.125 x 11 = 49.6719 cubic inches. When we fold the sheet of paper horizontally into fourths, we get a shorter, but fatter prism with a height of 8.5 inches and a square base measuring 11 ÷ 4 = 2.75 inches on the side. The volume of this shorter, fatter prism is 2.75 x 2.75 x 8.5 = 64.2813 cubic inches. There are 64.2813 – 49.6719 ≈ 14.6 cubic inches more volume in the shorter, fatter prism.

7. The volume of a cylinder is the area of the base times the height, or $V = \pi r^2 h$. The quarter has a radius of $\frac{15}{32}$ of an inch and a height of $\frac{1}{6}$ of an inch, so its volume is $\pi \times \left(\frac{15}{32}\right)^2 \times \frac{1}{6} = \frac{225}{16{,}384}\pi$ cubic inches. A coin with 4 times the volume would have $4 \times \frac{225}{16{,}384}\pi = \frac{225}{4096}\pi$ cubic inches. Since the radius of this other coin is $\frac{9}{16}$ of an inch, we can find the height as follows:

$$\frac{225}{4096}\pi = \pi \times \left(\frac{9}{16}\right)^2 \times h \Rightarrow \frac{256}{81} \times \frac{225}{4096} = h \Rightarrow h = \frac{225}{81 \times 4096} = \frac{225}{1296}$$

which is $\frac{25}{144}$ of an inch.

8. 11.5 cm

9. 3000 cm^3

10. 3281.25 cm^3

11. 324 cm^3

12. a) volume of the large cone = 83.78 in^3
volume of the small cone = 0.17 in^3
b) difference between the cones' volumes = 83.61 in^3
c) volume of the cylinder = 3.02 in^3
d) total volume = 86.63 in^3

13. $529\frac{7}{8}$ cubic inches
The diameter of a hose with a 1" outside diameter and 1/8 inch walls is
1 – 1/8 – 1/8 = ¾ inch.
Height: 1200 inches
Area of circular top: $3.14 \times \frac{3}{8} \times \frac{3}{8} = .4415625$ square inches
Volume = .4415625 x 1200 = 529.875 cubic inches

14. 7 feet
Maximum load: 80,000 pounds - 6500 pounds =73,500 pounds
Volume of box: 25 x 12 x 8 = 2400 cubic feet.
Weight of full load: 2400 cubic feet x 35 pounds = 84,000 pounds
Weight per foot in height: 84,000 ÷ 8 = 10,500 pounds.
73,500 allowable ÷ 10,500 per foot = 7 feet high

15. .04 centimeters
The volume of the sheet must equal 1 cubic centimeter.
1.25 x 20 x n = 1 cubic centimeter
25n = 1 cubic centimeter n = .04 centimeters thick

16. 7:04 P.M.
Change measurements to decimeters: Radius = 2.5 decimeters
Height: 15 decimeters
Area of top circular part: 3.14 x 2.5 x 2.5 = 19.625 square decimeters
Volume: 19.625 square decimeters x 15 (height) = 294.375 cubic decimeters
If it takes 6 seconds to fill one liter, it takes 6 x 294.375 = 1766.25 seconds to fill the tank.
1766.25 seconds ÷ 60 seconds in a minute = 29.4375 minutes
6:35 ÷ 29 = 7:04

17. 230 feet
The radius of Don's water heater is .75 feet
Change all measurements to inches: Height is 48 inches   Radius is 9 inches
Area of circular part of water heater: 3.14 x 9 x 9 = 254.34 square inches
Volume: 254.34 x 48 = 12,208.32 cubic inches
1/10 volume of water heater = 1220.832 cubic inches.

Radius of garden hose 3/8 or.375 inches
Area of circular part of hose: 3.14 x .375 x .375 = .4415625 square inches
Volume of needed hose: .4415625 square inches x *n* inches in length = 1220.832 cubic inches
.4415625n = 1220.832
Divide both sides by .4415625 : n = 2764.8 inches
2764.8 inches = 230.4 feet

18. 1. Steve or his dad could drink all of the water in the whole kiddie pool in about 2 years.
2. It would take Steve or his dad about 100 years to drink all of the water in the whole larger swimming pool.

1. First, I read in the problem that there are 7.48 gallons in a cubic foot of water. I decided to round this to 7.5 to make the calculations easier. Then I estimated that since his father would drink about a quart of water a day, Steve would drink about a quart of water a day also. Since there are 4 quarts in a gallon, they would each drink 1 gallon of water every 4 days. Since there are about 7.5 gallons in a cubic foot, you would multiply

7.5 gallons/cu ft x 4 days/gallon = 30 days/cu ft

So that means that it would take Steve or his dad 30 days to drink one cubic foot of water.

Next, I found out how many cubic feet of water were in Steve's kiddie pool. On the surface, the area was

5 ft x 5 ft = 25 sq ft.

Since the kiddie pool was one foot deep, I multiplied

25 sq ft x 1 ft = 25 cu ft.

To see how many days it would take them to drink all the water, I multiplied

25 cu ft x 30 days/cu ft = 750 days.

Since I rounded the gallons per cu ft up, and they probably would drink a little more in the summertime, I think it would take each about 2 years (730 days) to drink all the water in the kiddie pool.

2. For the larger pool, I needed to find out the cubic feet of water it held. I decided to round the 12 ft dimension to 10 ft, the 24 ft dimension to 25 ft, and the 4.5 ft dimension to 5 ft so the numbers would be easy to do mental calculations on. To find out how many cubic feet of water the pool would hold, I needed to multiply these three numbers together.

10 ft x 25 ft x 5 ft = (10 x 25 x 5) cubic feet.

Then I realized that for the kiddie pool, that held 25 cubic feet of water, it took Steve or his dad about 2 years to drink all the water. So, I know that every 2 years each can drink 25 cubic feet of water. So, the equations becomes

(10 x 25 x 5) cu ft / (25 cu ft/2 years) = how many years it would take each of them to drink all the water in the larger pool.

I could cancel out the 25s on top and bottom of the equation, and the cubic feet units on the top and bottom of the equations also cancel out.

That left me with (10 x 5)/(1/2) = (10 x 5) x 2 = 100 years to drink all of the water in the larger pool.

19. The Right Angles can fit more ice cream boxes in the freezer (total 20). The amount of unused space left in the freezer would be 52,843 cubic centimeters. Extra: A way to fit more whole containers of ice cream in the freezer would be to stack
First I found the volume of the freezer: 65cm x 45cm x 35cm - 102,375 cubic centimeters. Next I found the volume of one box/container: 17.5cm x 11.6cm. x 12.2cm = 2,476.6 cubic centimeters.

Working on the Protractors' method, I next figured out how many boxes of ice cream would fit across the width of the freezer: 65cm/17.5cm = 3.714. Because the boxes are whole items, this would round to 3 boxes across width. Then I found how many would fit going back deep into freezer: 45cm/11.6cm = 3.879 which again must be rounded down to 3. One layer of boxes would be 3 x 3 = 9. Next I found how many layers of boxes can be fit into height of freezer: 35cm/12.2cm = 2.868 which rounds down to 2. The one layer of 9 multiplied by 2 (layers) equals 18 boxes total with Protractors' method.

Working on the Right Angles' method, I figured out how many boxes of ice cream would fit across the width of the freezer: 65cm/11.6cm = 5.603 rounded to 5. Then I found how many would fit going back deep into the freezer: 45cm/17.5cm = 2.571 rounded to 2. One layer of boxes would be 5 x 2 = 10. Next I found how many layers can be fit into height of freezer: 35cm/12.2cm = 2.868 rounded to 2. One layer of 10 multiplied by 2 equals 20 boxes total with Right Angles' method.

To find out how much unused space would be left in freezer if they buy 20 boxes, I multiplied volume of single box by 20: 2,476.6 cm x 20 = 49,532 cubic centimeters. Then I subtracted this number from total volume of freezer: 102,375 cubic centimeters - 49,532 cubic centimeters = 52,843 cubic centimeters.

The Right Angles' method fit 2 more boxes than the Protractors' method: 20-18=2.

EXTRA: I thought that it would be space efficient if you would to stack unopened boxes that would not create a mess on their sides. I found how many would fit across using width of 11.6: 65cm/11.6cm = 5.603 rounded to 5. Then found how many fit depth of freezer: 45cm/12.2cm = 3.688 rounded to 3. One layer of boxes would be 5 x 3 = 15. (I also tried with 12.2cm and depth of 11.6 with same results.) I then found how many layers would fit height of freezer: 35cm/17.5cm = 2. Total of unopened boxes that could fit in freezer
would be 15 x 2 = 30.

However, if you were to take into account occurrence of open boxes, you would have to make one top layer with flaps on top to prevent a mess. I used the Right Angles method for a top layer for 10 boxes and side stack method for bottom layer to allow another 15 boxes for total of 25.

20. The ratio of your volume to TJs is 5 to 1.First I was about to find the volume when I got an idea that I didn't need to do that. I then converted the fraction that the water raised when TJ got in (3/5 of an inch) to a decimal. I did that by saying to myself that 3/5=6/10, and 6/10=60/100, or 0.60. After that I figured that the water rose 3 inches when you got in. So then I figured out that 0.6 goes into 3 5 times, because 6 goes into 30 5 times.
So that's the answer 5 to 1.

My answer would be the same if I found the volume of the pool because the width and the length don't change, so that doesn't make a difference in the answer. A volume is like the total space something takes up, you get volume by multiply the height by the width by the length, if the length and the width stay the same you don't have to use them. For Example 1/2 stays the same when both the numerator and the denominator are multiplied by the same number.

Bibliography Information

Teachers attempted to cite the sources for the problems included in this problem set. In some cases, sources may not have been known.

Problems	Bibliography Information	
18 - 20	The Math Forum @ Drexel (http://mathforum.org/)	
13 - 17	Zaccaro, Edward. Challenge Math (Second Edition): Hickory Grove Press, 2005.